FORMULAS FOR MATH

Name: ______________________________

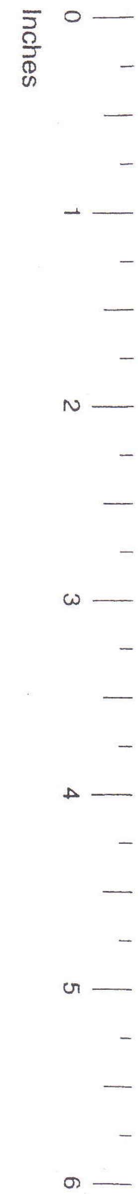
KEEP THIS FOR EVERY MATH SUBJECT.

1. Geometry
2. Algebra I and II
3. PRE-CALCULUS
4. CALCULUS.

Grades 9, 10, and 11 Exit Level Mathematics Chart

Perimeter	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	circle	$A = \pi r^2$
Surface Area	cube	$S = 6s^2$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$
Volume	prism or cylinder	$V = Bh$*
	pyramid or cone	$V = \frac{1}{3}Bh$*
	sphere	$V = \frac{4}{3}\pi r^3$
**B represents the area of the Base of a solid figure.*		
Pi	π	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$

Grades 9, 10, and 11 Exit Level
Mathematics Chart



LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

Continued on the next side

FORMULAS FROM GEOMETRY

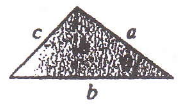
Triangle

$h = a \sin \theta$

Area $= \frac{1}{2}bh$

(Law of Cosines)

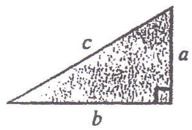
$c^2 = a^2 + b^2 - 2ab \cos \theta$



Right Triangle

(Pythagorean Theorem)

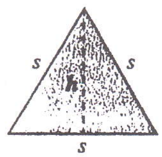
$c^2 = a^2 + b^2$



Equilateral Triangle

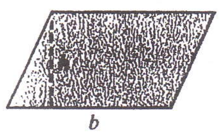
$h = \frac{\sqrt{3}s}{2}$

Area $= \frac{\sqrt{3}s^2}{4}$



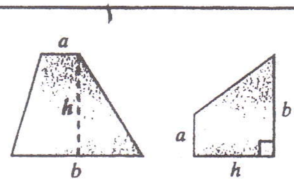
Parallelogram

Area $= bh$



Trapezoid

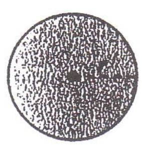
Area $= \frac{h}{2}(a + b)$



Circle

Area $= \pi r^2$

Circumference $= 2\pi r$

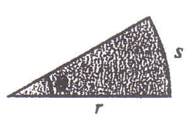


Sector of Circle

(θ in radians)

Area $= \frac{\theta r^2}{2}$

$s = r\theta$



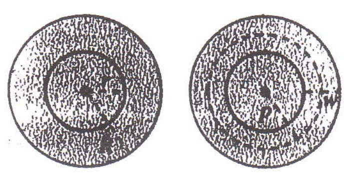
Circular Ring

(p = average radius,

w = width of ring)

Area $= \pi(R^2 - r^2)$

$= 2\pi pw$



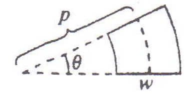
Sector of Circular Ring

(p = average radius,

w = width of ring,

θ in radians)

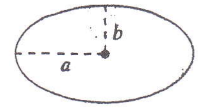
Area $= \theta pw$



Ellipse

Area $= \pi ab$

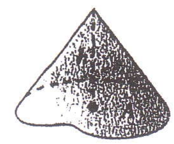
Circumference $\approx 2\pi \sqrt{\frac{a^2 + b^2}{2}}$



Cone

(A = area of base)

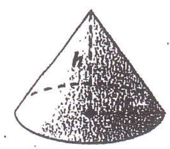
Volume $= \frac{Ah}{3}$



Right Circular Cone

Volume $= \frac{\pi r^2 h}{3}$

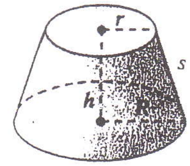
Lateral Surface Area $= \pi r \sqrt{r^2 + h^2}$



Frustum of Right Circular Cone

Volume $= \frac{\pi(r^2 + rR + R^2)h}{3}$

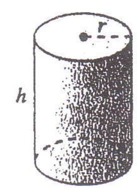
Lateral Surface Area $= \pi s(R + r)$



Right Circular Cylinder

Volume $= \pi r^2 h$

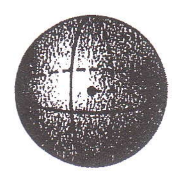
Lateral Surface Area $= 2\pi rh$



Sphere

Volume $= \frac{4}{3}\pi r^3$

Surface Area $= 4\pi r^2$

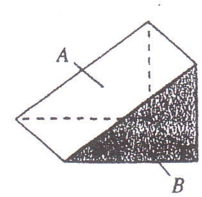


Wedge

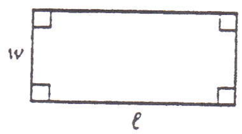
(A = area of upper face,

B = area of base)

$A = B \sec \theta$



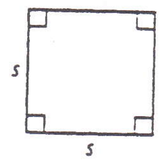
GEOMETRIC FORMULAS



Rectangle

Area: $A = \ell w$

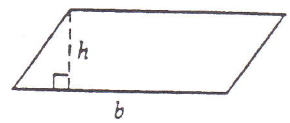
Perimeter: $p = 2\ell + 2w$



Square

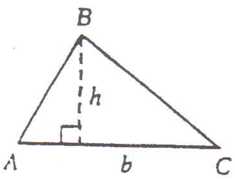
Area: $A = s^2$

Perimeter: $p = 4s$



Parallelogram

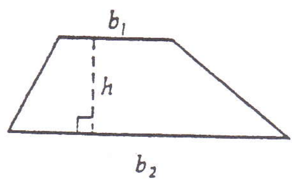
Area: $A = bh$



Triangle

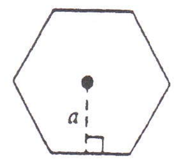
Area: $A = \frac{1}{2}bh$

$m\angle A + m\angle B + m\angle C = 180°$



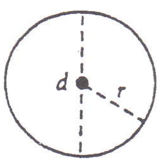
Trapezoid

Area: $A = \frac{1}{2}h(b_1 + b_2)$



Regular Polygon

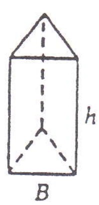
Area: $A = \frac{1}{2}ap$



Circle

Area: $A = \pi r^2$

Circumference: $C = \pi d = 2\pi r$

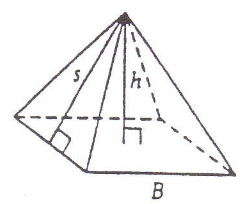


Right Prism

Volume: $V = Bh$

Lateral Area: $LA = ph$

Surface Area: $SA = ph + 2B$

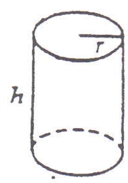


Regular Pyramid

Volume: $V = \frac{1}{3}Bh$

Lateral Area: $LA = \frac{1}{2}ps$

Surface Area: $SA = \frac{1}{2}ps + B$

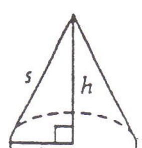


Right Cylinder

Volume: $V = \pi r^2 h$

Lateral Area: $LA = 2\pi rh$

Surface Area: $SA = 2\pi rh + 2\pi r^2$

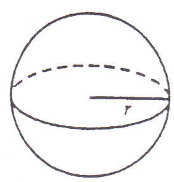


Right Cone

Volume: $V = \frac{1}{3}\pi r^2 h$

Lateral Area: $LA = \pi rs$

Surface Area: $SA = \pi rs + \pi r^2$



Sphere

Volume: $V = \frac{4}{3}\pi r^3$

Surface Area: $SA = 4\pi r^2$

$\sec\theta = \frac{r}{x} = \frac{hyp}{adj}$ $\cot\theta = \frac{x}{y} = \frac{adj}{opp}$

$\tan\theta = \frac{y}{x} = \frac{opp}{adj}$ $\csc\theta = \frac{r}{y} = \frac{hyp}{opp}$

$\sin\theta = \frac{y}{r} = \frac{opp}{hyp}$ $\cos\theta = \frac{x}{r} = \frac{adj}{hyp}$

$\text{distance} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\text{midpoint}\ \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

30-60-90 rt. Δ

shorter leg = $\frac{hyp}{2}$

longer leg = $\sqrt{3}$(shorter leg)

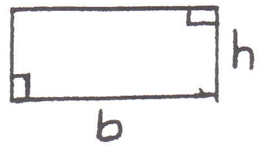
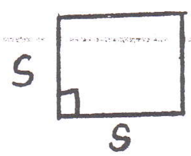
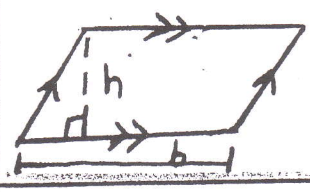
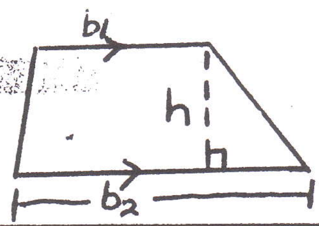
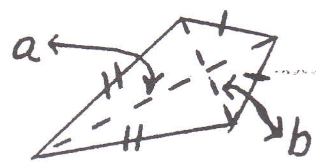
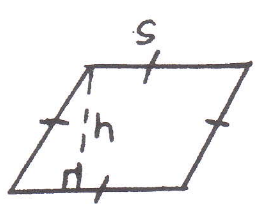
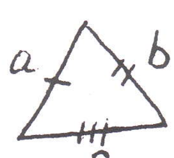
45-45-90 rt. Δ

hyp = $\sqrt{2}$(leg)

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Areas Of Quads

NAME	Drawing	FORMULAS	Description
Rectangle		$A=bh$	
Square		$A=s^2$	
Parallelogram		$A=bh$	
Trapezoid		$A=\frac{1}{2}(b_1+b_2)h$	
Kite		$A=\frac{ab}{2}$	a & b stand for diagonal
Rhombus		$A=sh$ or $A=\frac{ab}{2}$	
* HERON'S FORMULA *	 $S=\frac{1}{2}(a+b+c)$	$A=\sqrt{s(s-a)(s-b)(s-c)}$	

Properties for Proofs

Text book page: 119

REFLEXIVE PROPERTY

$a = a$

SYMMETRIC PROPERTY

If $a = b$, then $b = a$

TRANSITIVE PROPERTY

If $a = b$ and $b = c$, then $a = c$

ADDITION PROPERTY

If $a = b$, then

$a + c = b + c$

SUBTRACTION PROPERTY

If $a = b$, then

$a - c = b - c$

SUBSTITUTION PROPERTY

If $a = b$, then **a** can be substituted for **b** (and **b** for **a**) in an expression.

If $a + c = 5$

and $a = b$, then

$b + c = 5$.

FINITE DIFFERENCE RULE

1. QUADRATIC EXPRESSION (QUADRATIC PATTERNS)

$$an^2 + bn + c$$

TERM	VALUE	D1	D2
0	c=		
1	6	a + b=	a=(D2)/2
2	28		a=(D2)/2
3	65		a=(D2)/2
4	117		a=(D2)/2
5	184		a=(D2)/2
6	266		a=(D2)/2
n			

2.LINEAR EXPRESSION (LINEAR PATTERNS)

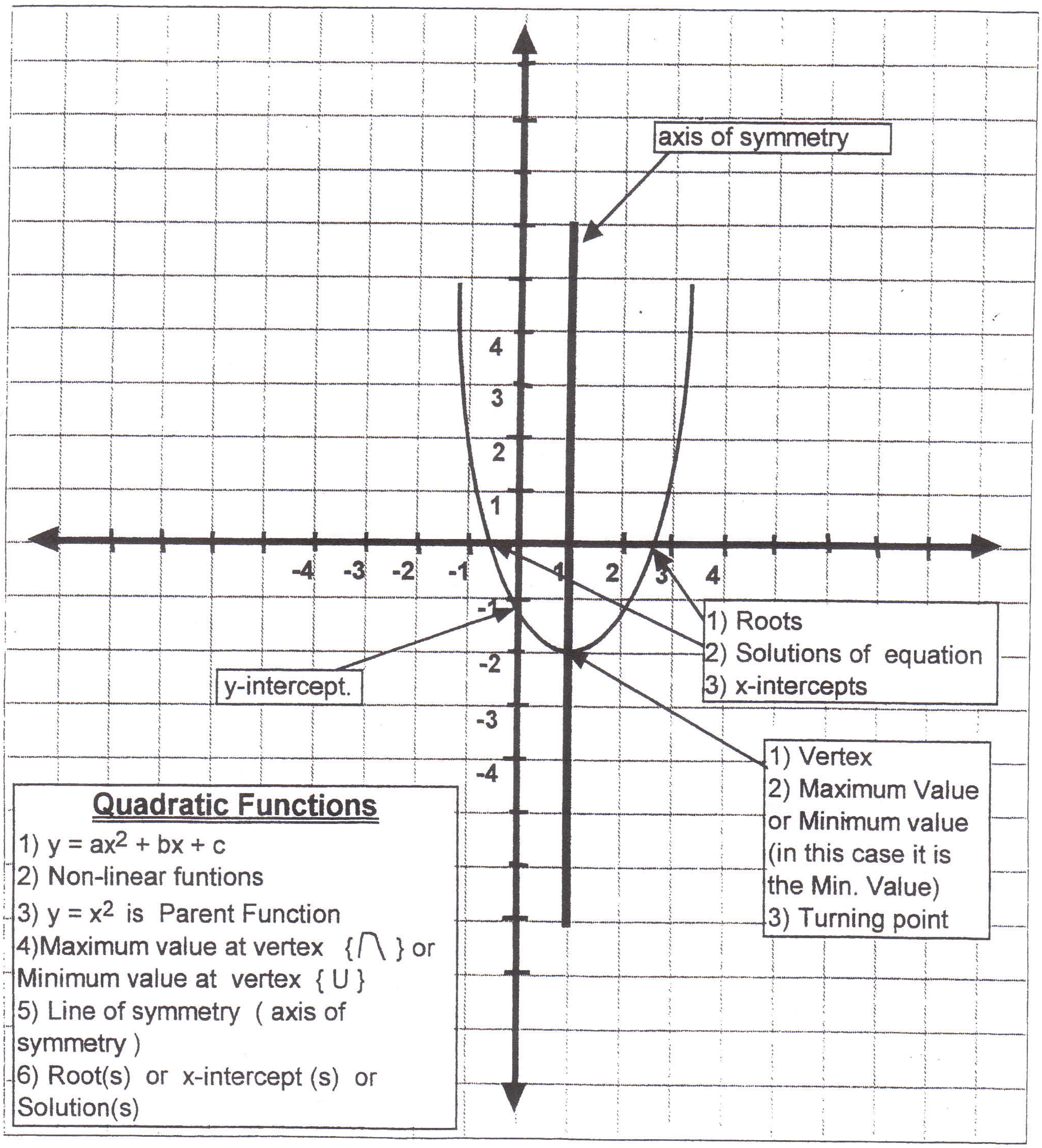
$$an + b$$

TERM	VALUE	D1	
0	b =		
1	-1	a= D1	
2	-6	a= D1	
3	-11	a= D1	
4	-16	a= D1	
5	-21	a= D1	
6	-26	a= D1	
n			

;QUADRATIC FUNCTION GRAPH COMPONENTS:

NAME: $y = ax^2 + bx + c$;

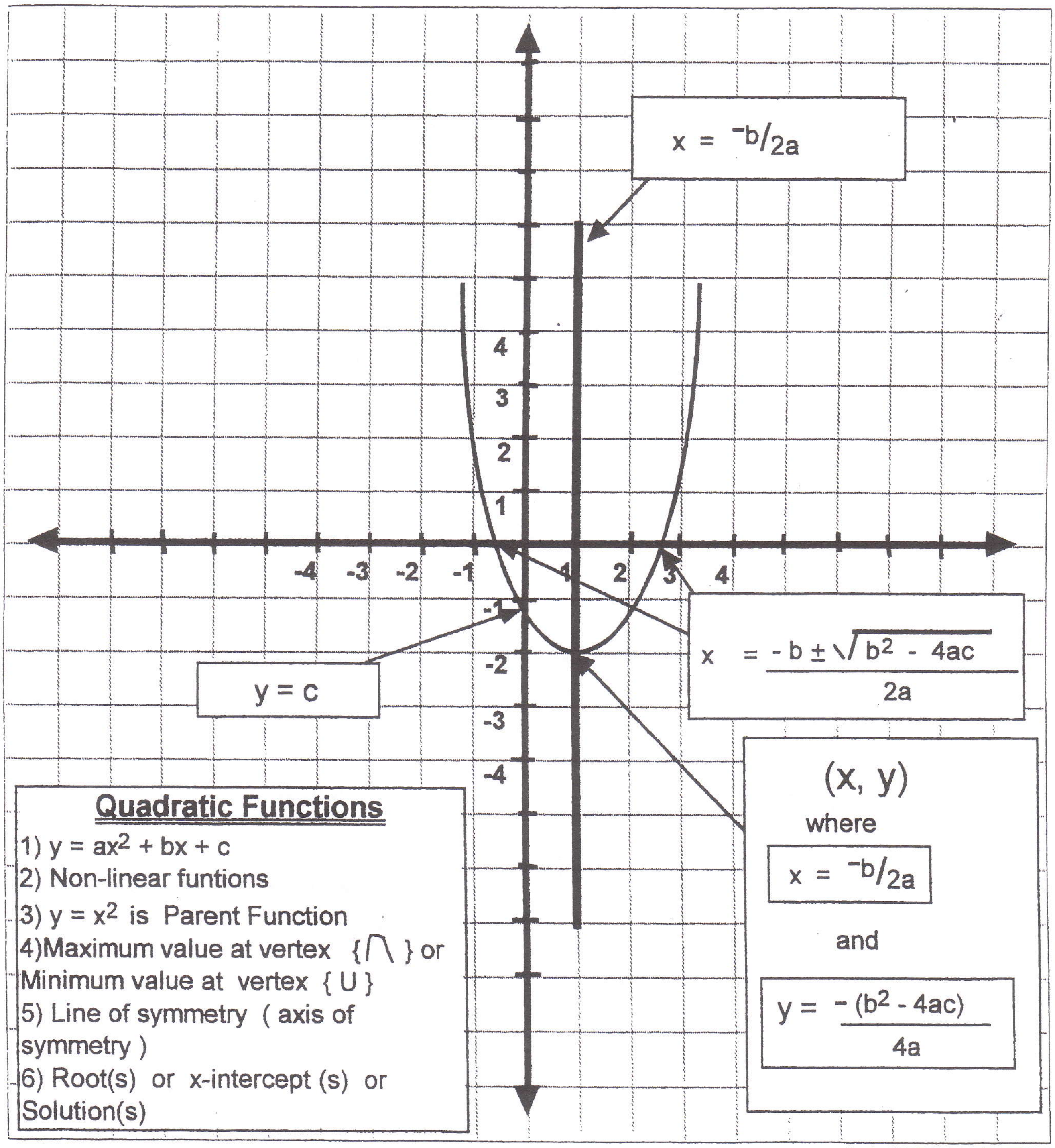
If $a < 0$, then you have a Maximum Value graph. If $a > 0$, Then you have Minimum Value graph. If $a = 0$, then you have a linear graph not a quadratic anymore. c is always the y-intercept.



;QUADRATIC GRAPH COMPONENTS EQUATIONS:

NAME: $y = ax^2 + bx + c$;

If $a < 0$, then you have a Maximum Value graph. If $a > 0$, Then you have Minimum Value graph. If $a = 0$, then you have a linear graph not a quadratic anymore. c is always the y-intercept.



Quadratic Functions

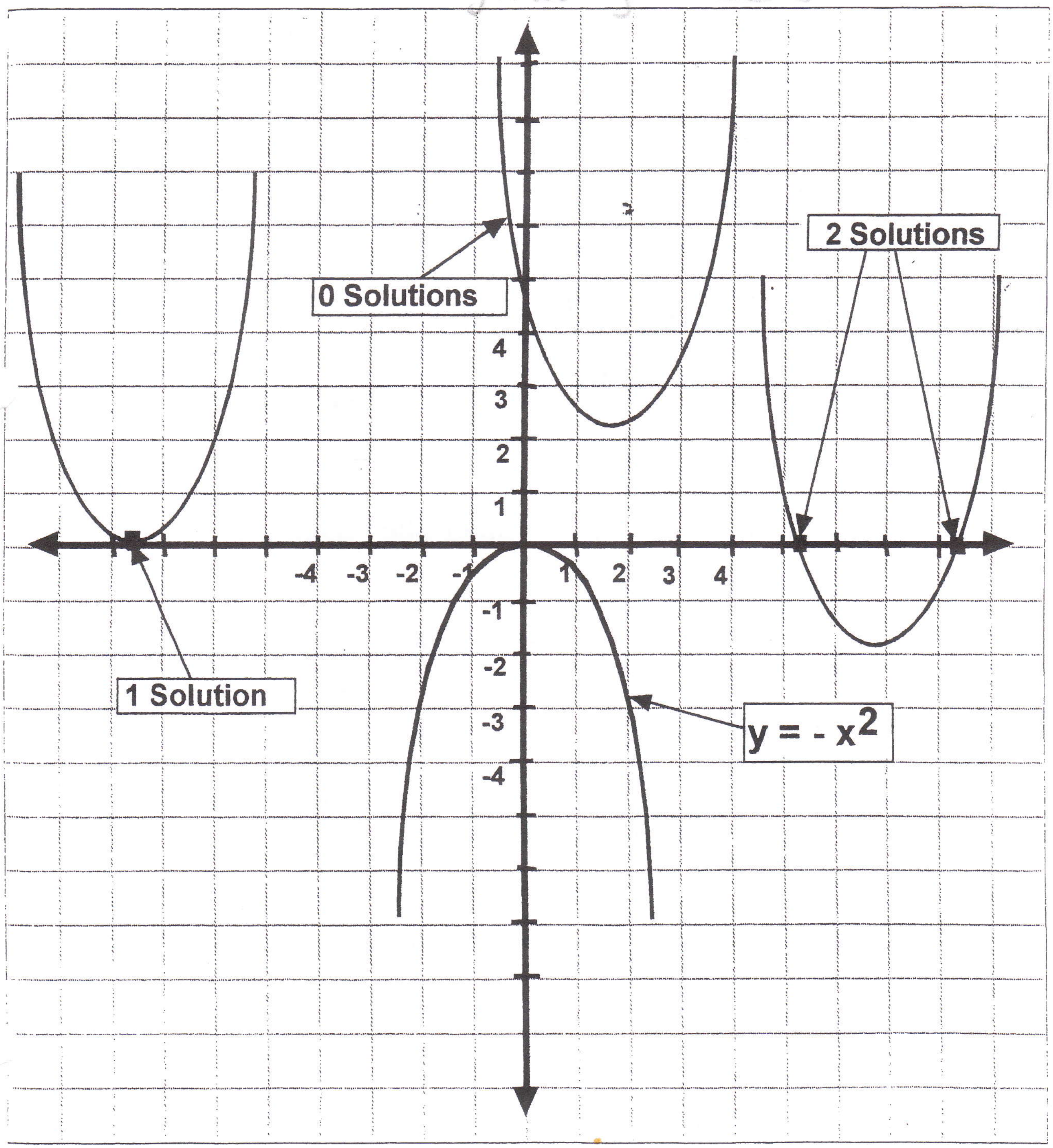
1) $y = ax^2 + bx + c$
2) Non-linear funtions
3) $y = x^2$ is Parent Function
4)Maximum value at vertex { ∩ } or Minimum value at vertex { U }
5) Line of symmetry (axis of symmetry)
6) Root(s) or x-intercept (s) or Solution(s)

;QUADRATIC FUNCTION GRAPH PROPERTIES:

NAME: $y = ax^2 + bx + c$;

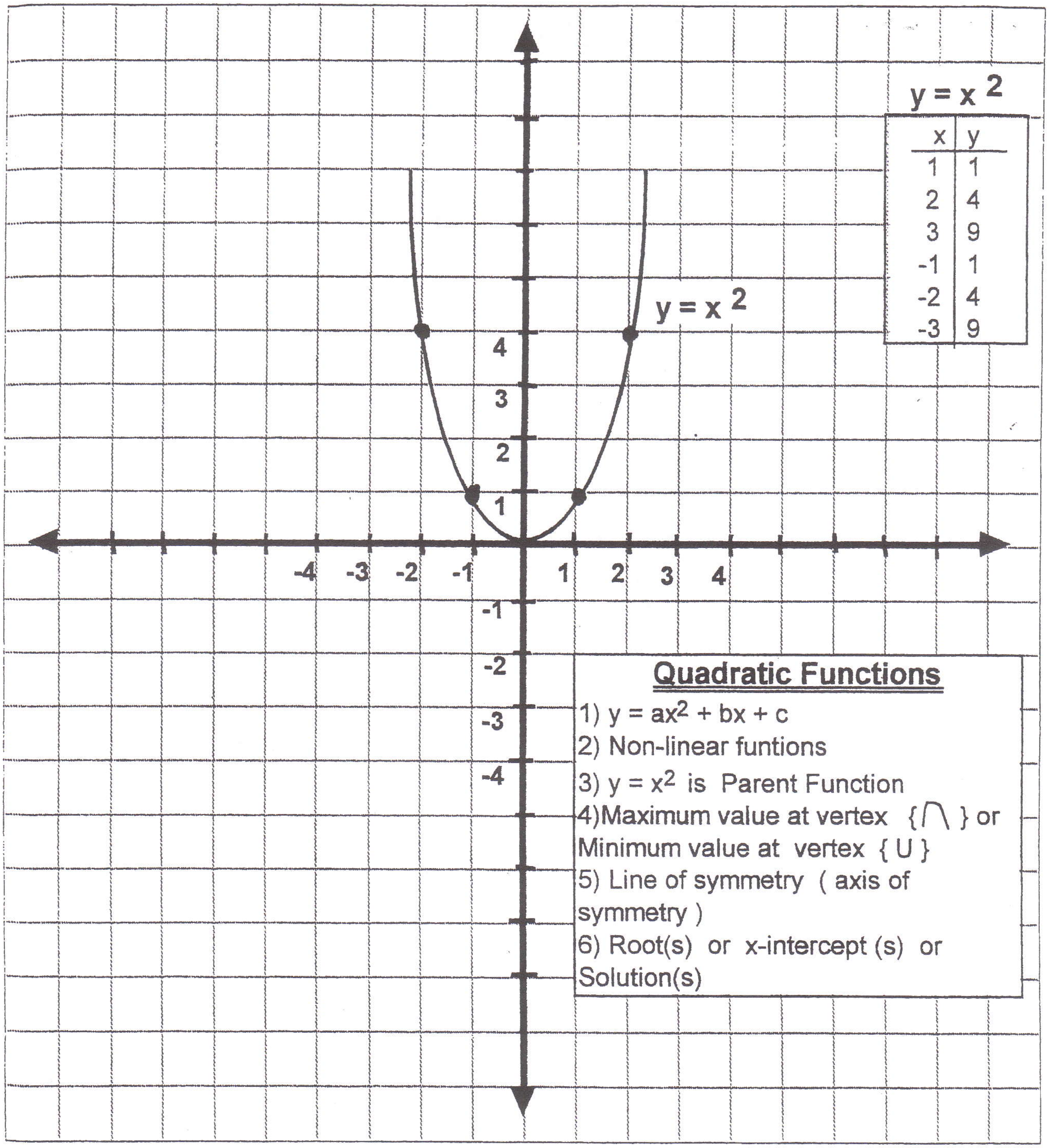
If a < 0, then you have a Maximum Value graph. If a >0, Then you have Minimum Value graph. If a = 0, then you have a linear graph not a quadratic anymore. c is always the y-intercept.

0 solutions = imaginary roots



;QUADRATIC PARENT FUNCTION :

NAME: ______________________________; **PERIOD:** ____.



$y = x^2$

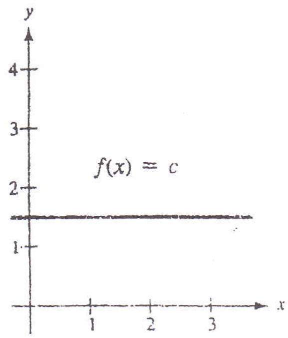
x	y
1	1
2	4
3	9
-1	1
-2	4
-3	9

Quadratic Functions

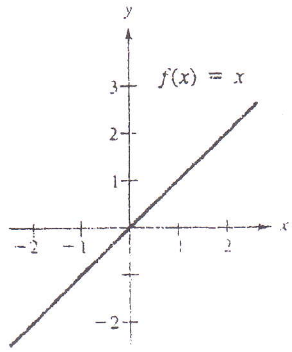
1) $y = ax^2 + bx + c$
2) Non-linear funtions
3) $y = x^2$ is Parent Function
4)Maximum value at vertex { ∩ } or Minimum value at vertex { U }
5) Line of symmetry (axis of symmetry)
6) Root(s) or x-intercept (s) or Solution(s)

GRAPHS OF COMMON FUNCTIONS

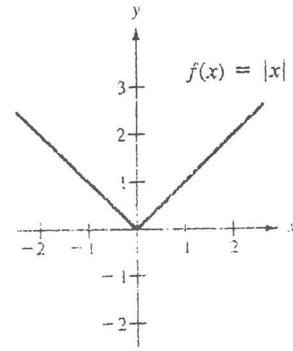
PARENT FUNCTIONS



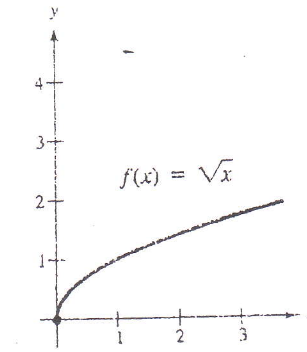
Constant Function



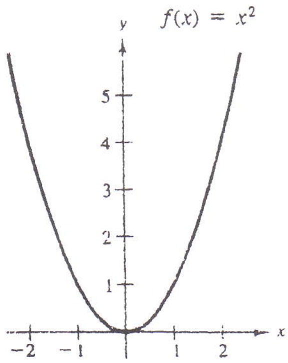
Identity Function



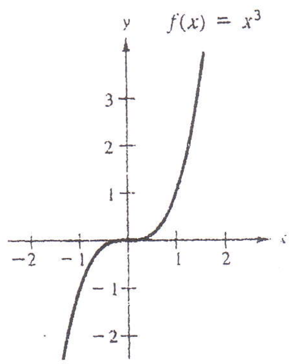
Absolute Value Function



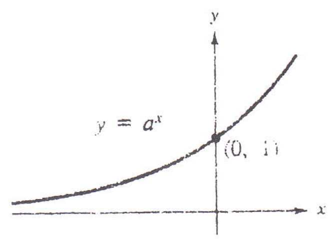
Square Root Function



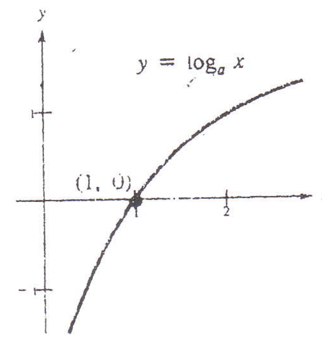
Squaring Function



Cubing Function



Exponential Function

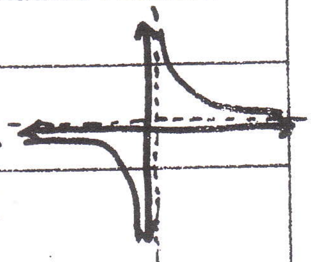


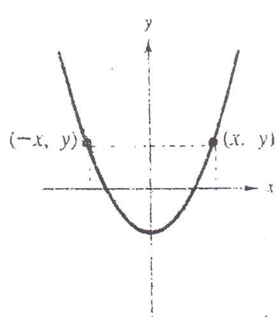
Logarithmic Function

SYMMETRY

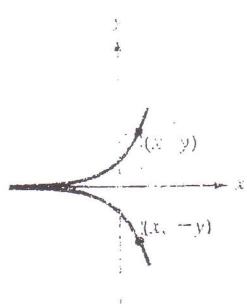
Rational Function

$f(x) = \frac{1}{x}$

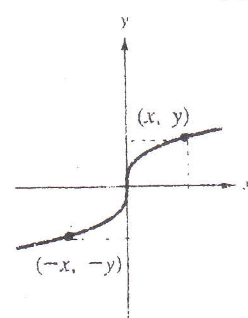




***y*-Axis Symmetry**



***x*-Axis Symmetry**



Origin Symmetry

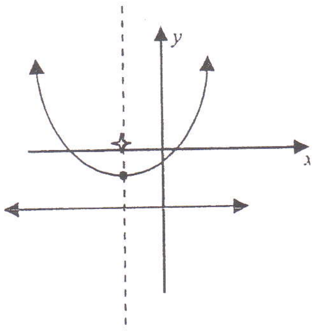
CONICS

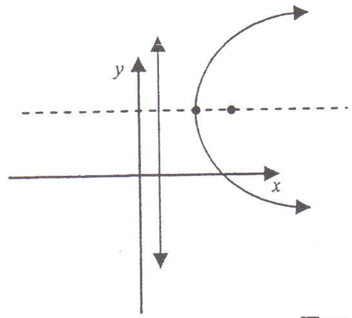
PARABOLA

$$y-k=a(x-h)^2$$

$$x-h=a(y-k)^2$$

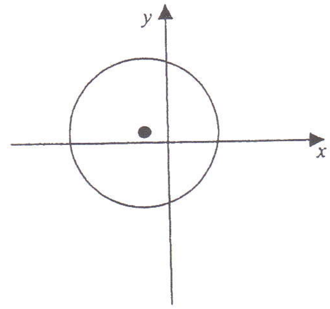
$$a=\frac{1}{4p}$$





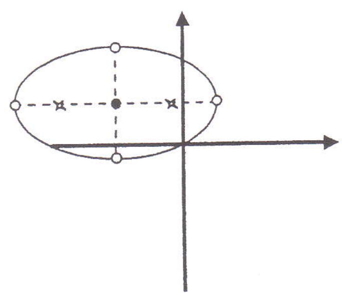
CIRCLE

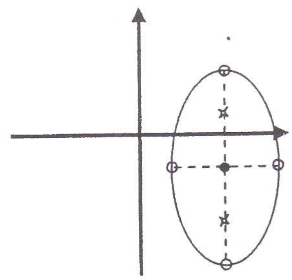
$$(x-h)^2+(y-k)^2=r^2$$



ELLIPSE

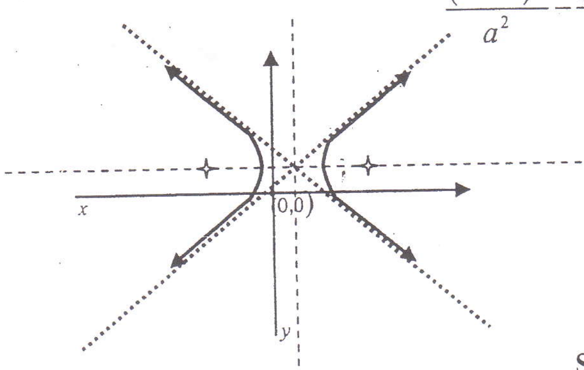
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

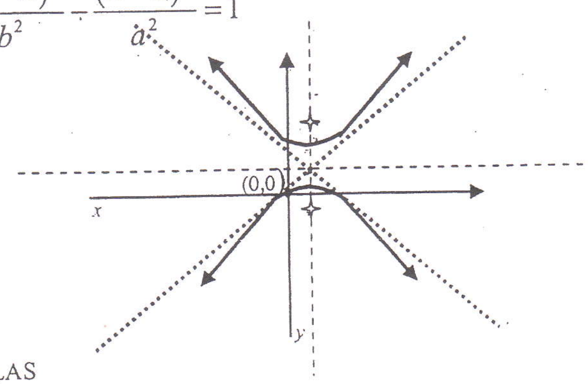




HYPERBOLA

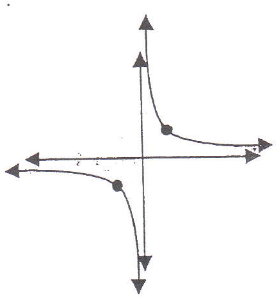
$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \textbf{ or } \frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$$

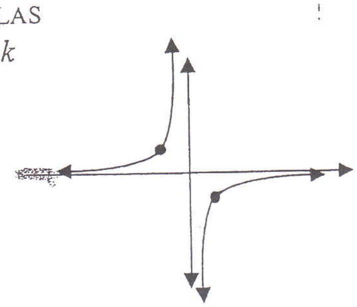




SPECIAL HYPERBOLAS

$xy=k$ **or** $xy=-k$

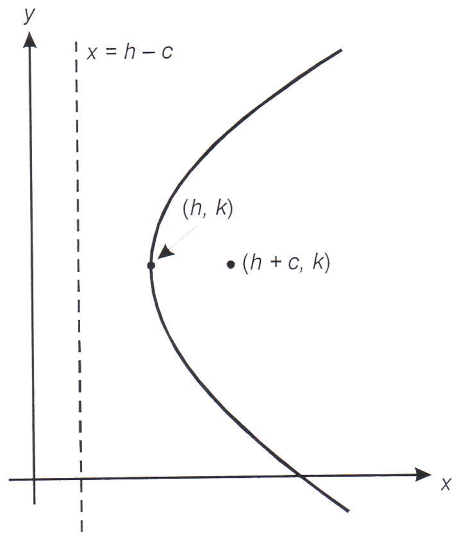




Definitions and Formulas for Use on Mathematics Items

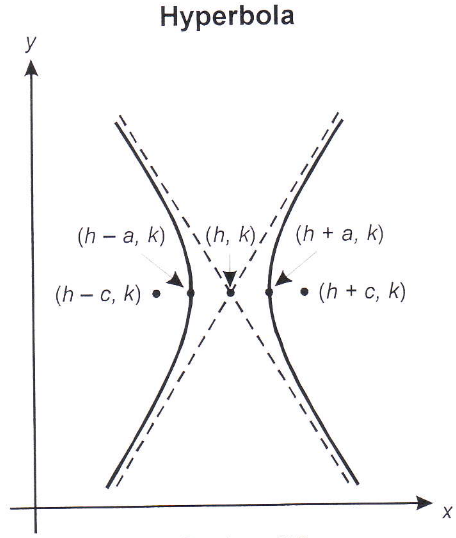
GEOMETRY

Parabola



$(y - k)^2 = 4c(x - h)$, where $c > 0$

Hyperbola



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1,$$

where $b^2 = c^2 - a^2$

ALGEBRA

For $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $(a \neq 0)$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest, where A is the final value, P is the principal, r is the interest rate, t is the term, n is divisions within the term

$[x] = n$ Greatest integer function, where n is the integer such that $n \leq x < n + 1$

VOLUME

Cylinder: (area of base) × height

Cone: $\frac{1}{3}$ (area of base) × height

Sphere: $\frac{4}{3}\pi$ (radius)^3

Prism: (area of base) × height

AREA

Triangle: $\frac{1}{2}$ base × height

Rhombus: $\frac{1}{2}$ diagonal$_1$ × diagonal$_2$

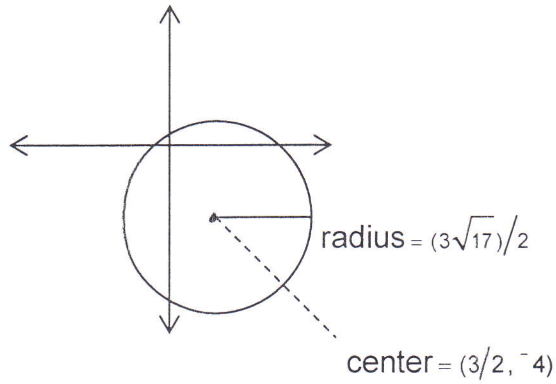
Trapezoid: $\frac{1}{2}$ height (base$_1$ + base$_2$)

Sphere: 4π (radius)^2

Circle: π (radius)^2

Lateral surface area of cylinder: 2π (radius) × height

Graph the circle.



To write the equation given the center and the radius use the standard form of the equation of a circle:

$$(x-h)^2+(y-k)^2=r^2$$

Sample problems:

Given the center and radius, write the equation of the circle.

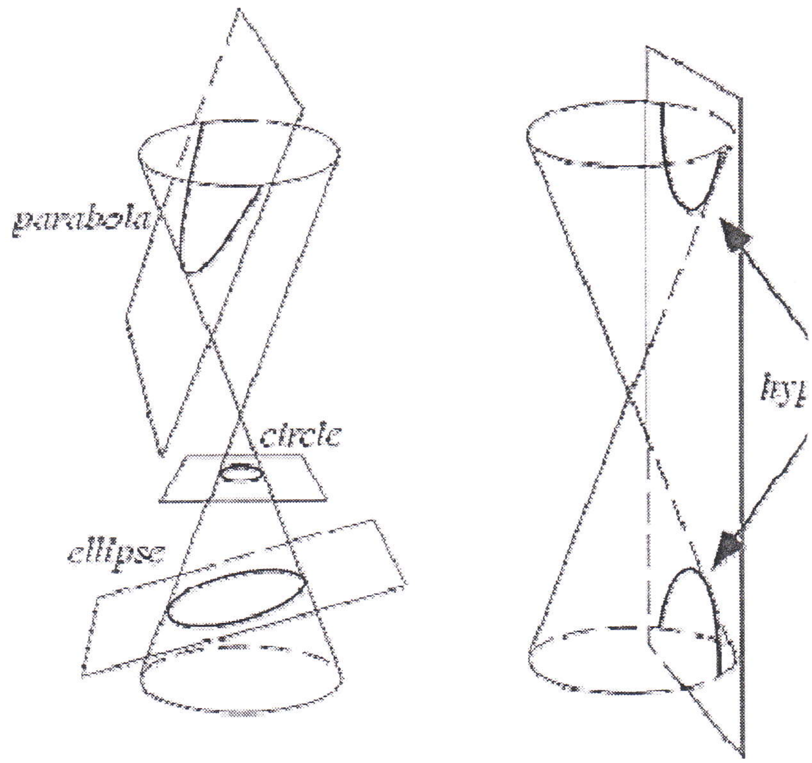
1. Center $({}^-1,4)$; radius 11

$(x-h)^2+(y-k)^2=r^2$	1. Write standard equation.
$(x-({}^-1))^2+(y-(4))^2=11^2$	2. Substitute.
$(x+1)^2+(y-4)^2=121$	3. Simplify.

2. Center $(\sqrt{3}, {}^-1/2)$; radius $= 5\sqrt{2}$

$(x-h)^2+(y-k)^2=r^2$	1. Write standard equation.
$(x-\sqrt{3})^2+(y-({}^-1/2))^2=(5\sqrt{2})^2$	2. Substitute.
$(x-\sqrt{3})^2+(y+1/2)^2=50$	3. Simplify.

Conic sections result from the intersection of a cone and a pla
The three main types of conics are parabolas, ellipses, and hyperbolas.



The general equation for a conic section is:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

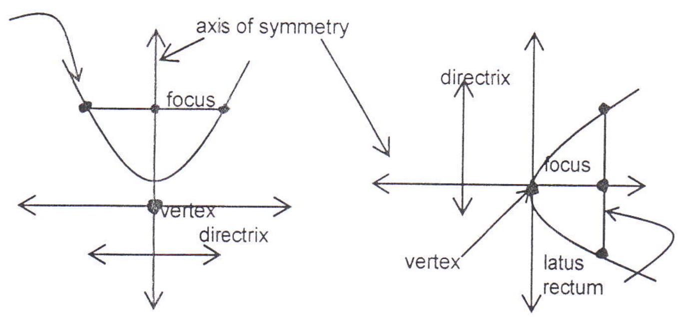
The value of $B^2 - 4AC$ determines the type of conic. If $B^2 - 4A$ less than zero the curve is an ellipse or a circle. If equal to zer the curve is a parabola. If greater than zero, the curve is a hyperbola.

PARABOLAS

-A parabola is a set of all points in a plane that are equidistant from a fixed point (focus) and a line (directrix).

FORM OF EQUATION	$y = a(x-h)^2 + k$	$x = a(y-k)^2 + h$
IDENTIFICATION	x^2 term, y not squared	y^2 term, x not squared

SKETCH OF GRAPH



AXIS OF SYMMETRY	$x = h$	$y = k$

-A line through the vertex and focus upon which the parabola is symmetric.

VERTEX	(h,k)	(h,k)

-The point where the parabola intersects the axis of symmetry.

FOCUS	$(h, k + 1/4a)$	$(h + 1/4a, k)$
DIRECTRIX	$y = k - 1/4a$	$x = h - 1/4a$
DIRECTION OF OPENING	up if $a > 0$, down if $a < 0$	right if $a > 0$, left if $a < 0$
LENGTH OF LATUS RECTUM	$\lvert 1/a \rvert$	$\lvert 1/a \rvert$

-A chord through the focus, perpendicular to the axis of symmetry, with endpoints on the parabola.

Sample Problem:

1. Find all identifying features of $y = {}^-3x^2 + 6x - 1$.

First, the equation must be put into the general form $y = a(x - h)^2 - k$.

$$y = {}^-3x^2 + 6x - 1$$
$$= {}^-3(x^2 - 2x + 1) - 1 + 3$$
$$= {}^-3(x - 1)^2 + 2$$

1. Begin by completing the square.

2. Using the general form of the equation begin to identify known variables.

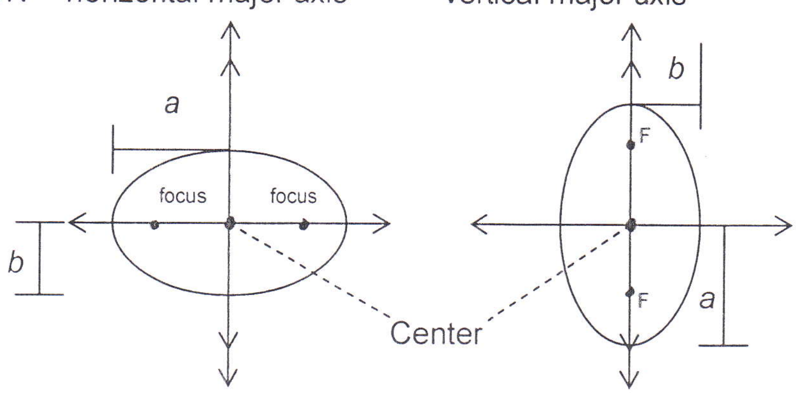
$a = {}^-3 \quad h = 1 \quad k = 2$

axis of symmetry: $x = 1$
vertex: (1,2)
focus: $(1, 1\tfrac{1}{4})$
directrix: $y = 2\tfrac{3}{4}$
direction of opening: down since $a < 0$
length of latus rectum: 1/3

ELLIPSE

FORM OF EQUATION	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
(for ellipses where $a^2 > b^2$).	where $b^2 = a^2 - c^2$	where $b^2 = a^2 - c^2$
IDENTIFICATION	horizontal major axis	vertical major axis

SKETCH



CENTER	(h,k)	(h,k)
FOCI	$(h \pm c, k)$	$(h, k \pm c)$
MAJOR AXIS LENGTH	$2a$	$2a$
MINOR AXIS LENGTH	$2b$	$2b$

Sample Problem:

Find all identifying features of the ellipse $2x^2 + y^2 - 4x + 8y - 6 = 0$. First, begin by writing the equation in standard form for an ellipse.

$$2x^2 + y^2 - 4x + 8y - 6 = 0$$

1. Complete the square for each variable.

$$2(x^2 - 2x + 1) + (y^2 + 8y + 16) = 6 + 2(1) + 16$$

$$2(x-1)^2 + (y+4)^2 = 24$$

2. Divide both sides by 24.

$$\frac{(x-1)^2}{12} + \frac{(y+4)^2}{24} = 1$$

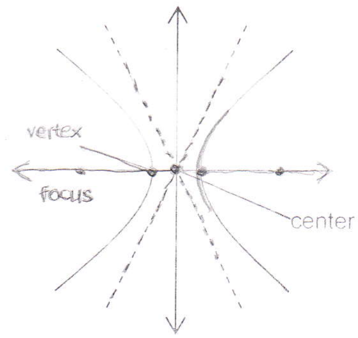
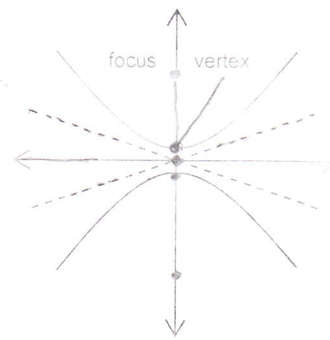
3. Now the equation is in standard form.

Identify known variables: $h = 1 \quad k = {}^{-}4 \quad a = \sqrt{24}$ or $2\sqrt{6}$
$b = \sqrt{12}$ or $2\sqrt{3} \quad c = 2\sqrt{3}$

Identification: vertical major axis
Center: $(1, {}^{-}4)$
Foci: $(1, {}^{-}4 \pm 2\sqrt{3})$
Major axis: $4\sqrt{6}$
Minor axis: $4\sqrt{3}$

HYPERBOLA

FORM OF EQUATION	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ where $c^2 = a^2 + b^2$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ where $c^2 = a^2 + b^2$

IDENTIFICATION	horizontal transverse axis (y^2 is negative)	vertical transverse axis (x^2 is negative)
SKETCH		
SLOPE OF ASYMPTOTES	$\pm(b/a)$	$\pm(a/b)$
TRANSVERSE AXIS (endpoints are vertices of the hyperbola and goes through the center)	$2a$ -on y axis	$2a$ -on x axis
CONJUGATE AXIS (perpendicular to transverse axis at center)	$2\,b$, -on y axis	$2b$, -on x axis
CENTER FOCI VERTICES	(h,k) $(h \pm c,k)$ $(h \pm a,k)$	(h,k) $(h,k \pm c)$ $(h,k \pm a)$

Sample Problem:

Find all the identifying features of a hyperbola given its equation.

$$\frac{(x+3)^2}{4} - \frac{(y-4)^2}{16} = 1$$

Identify all known variables: $h = {}^{-}3 \quad k = 4 \quad a = 2 \quad b = 4$

$c = 2\sqrt{5}$

Slope of asymptotes: $\pm 4/2$ or ± 2

Transverse axis: 4 units long

Conjugate axis: 8 units long

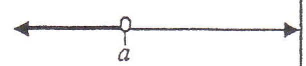
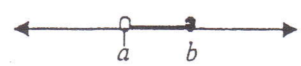
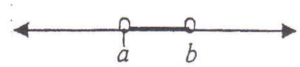
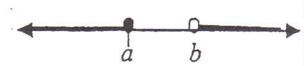
Center: $({}^{-}3,4)$

Foci: $({}^{-}3 \pm 2\sqrt{5},4)$

Vertices: $({}^{-}1,4)$ and $({}^{-}5,4)$

Interval notation is used in Algebra II for many topics.

1. Symbols:
    a. ∞ infinity
    b. $-\infty$ negative infinity
    c. [or] the value is included
    d. (or) the value is not included
    e. $\cup$ union or joining together two sets of numbers
2. Infinity **always** using soft brackets.

Interval Notation	*Set Builder Notation*	*Inequality Notation*	*Graph of Inequality*
$(-\infty, a]$	$\{x \mid x \le a\}$	$x \le a$	
$(-\infty, a)$	$\{x \mid x < a\}$	$x < a$	
$[b, \infty)$	$\{x \mid x \ge b\}$	$x \ge b$	
(b, ∞)	$\{x \mid x > b\}$	$x > b$	
$[a, b]$	$\{x \mid a \le x \le b\}$	$a \le x \le b$	
$[a, b)$	$\{x \mid a \le x < b\}$	$a \le x < b$	
$(a, b]$	$\{x \mid a < x \le b\}$	$a < x \le b$	
(a, b)	$\{x \mid a < x < b\}$	$a < x < b$	
$(-\infty, a] \cup (b, \infty)$	$\{x \mid x \le a \text{ or } x > b\}$	$x \le a, x > b$	

Transformations for y = f(x)

$y = f(x) + k$	vertical shift k units up
$y = f(x) - k$	vertical shift k units down
$y = f(x - h)$	horizontal shift h units right
$y = f(x + h)$	horizontal shift h units left
$y = -f(x)$	reflected in the x-axis
$y = f(-x)$	reflected in the y-axis
$x = f(y)$	reflected in the line y = x
$y = \|f(x)\|$	unchanged when f(x) > 0; reflected in the x-axis when f(x) < 0
$y = f(\|x\|)$	unchanged when x > 0; reflected in the y-axis when x < 0
$y = af(x), \|a\| > 1$	stretched vertically
$y = af(x), 0 < \|a\| < 1$	shrunk vertically
$y = f(bx), \|b\| > 1$	shrunk horizontally
$y = f(bx), 0 < \|b\| < 1$	stretched horizontally

multiply "y" values by "a"

multiply "x" values by $\frac{1}{b}$

Vertical stretch or shrink; reflection across x-axis

Vertical shift

$$y = a\,f(b(x+c))+d$$

Horizontal stretch or shrink; reflection across y-axis

Horizontal shift

Notes: Exponentiation

Definitions:

An **exponential function** is a function whose general equation is $y = a \cdot b^x$ where *a* and *b* stand for constants, b is positive and *x* and *y* are independent and dependent variables.

Exponentiation for positive integer exponent: (when *n* is positive) x^n means the product of *n* *x*'s.

In the expression x^n,

x is called the *base,*
n is called the *exponent*
x^n is called a *power.*

Example 1: $-x^3 = -(x)(x)(x)$
$(-x)^3 = (-x)(-x)(-x)$

Example 2: $4x^4 = 4(x)(x)(x)(x)$

$(4x)^4 = (4x)(4x)(4x)(4x)$

Rule: If the *base* contains more than one symbol, then the *base* must be placed in <u>parentheses</u>.

Properties of Exponentiation:

1. *Product of two powers with equal bases:*

$$x^a \cdot x^b = x^{a+b}$$

2. *Quotient of two powers with equal bases:*

$$\frac{x^a}{x^b} = x^{a-b}$$

3. *Power of a power:*

$$(x^a)^b = x^{a * b}$$

Log Rules

~~X~~

$a = b^c \Rightarrow \log_b a = c$

Standard Form, Base, Exponent

standard, base, Exponent

① $a = b^c \Rightarrow \log_b a = c$

② $\log_b MN \Rightarrow \log_b M + \log_b N$

③ $\log_b \frac{M}{N} \Rightarrow \log_b M - \log_b N$

④ $\log_b N^a \Rightarrow a \log_b N$

⑤ $\log_{b^*} a = \dfrac{\log a}{\log b}$

eg: $\log_5 3 = \dfrac{\log 3}{\log 5}$ use calculator Now.

Sequences and Series

① Sequence: A Sequence set of numbers which Follow on one another in such a manner that each number of the Sequence may be found from the preceding one by some fixed Rule or law

$\{1, 2, 3, \ldots n\}$

$\{a_1, a_2, a_3, \ldots a_n\}$

② Finite sequence: ⇒ set number of terms (n^{th} terms)

Infinite sequence ⇒ the number of terms is infinitely large.

④ Series: $a_1 + a_2 + a_3 + a_4 \ldots$

⑤ Arithmetic → series: 3, 8, 13, 18

Arithmetic → Sequence {3, 8, 13, 18, …}

Common difference ⇒ $d = a_2 - a_1$

⑥ Geometric → Series ⇒ $2 + 4 + 8 + 16 + \ldots$

Geometric → sequence ⇒ $2, 4, 8, 16, \ldots$

Common Ratio $(r) = \frac{a_2}{a_1}$

~~Form~~ Formulas

(1) $S_n = \dfrac{a(r^n-1)}{r-1}$ for $r>1$

(2) $S_n = \dfrac{a(1-r^n)}{1-r}$ for $r<1$

(3) Geometric Mean (x): between two numbers a and b

$$\Rightarrow \frac{x}{a} = \frac{b}{x} \Rightarrow x^2 = ab$$

$$\therefore \boxed{x = \sqrt{ab}}$$

(5) Geometric Sequence [Common Ratio (r)]

$a_1, a_1 r, a_1 r^2, a_1 r^3, a_1 r^4$

(6) Arithmetic Sequence [Common difference (d)]

$a_1, a_1+d, a_1+2d, a_1+3d, \ldots$

(7) Arithmetic Mean (x) of a and b

$$x = \frac{1}{2}(a+b)$$

Formulas

Arithmetic Sequences:

$$a_n = a_1 + (n-1)d$$

$$d = a_{n+1} - a_n$$

Arithmetic Series

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_n = \frac{n[2a_1 + (n-1)d]}{2}$$

Infinite Geometric Series

$$S = \frac{a_1}{1-r}, |r| < 1$$

Geometric Sequence

$$r = \frac{a_n}{a_{n-1}}; a_n = a_1 r^{n-1}$$

Geometric Series

$$S_n = \frac{a_1(1-r^n)}{1-r}; S_n = \frac{a_1 - r a_n}{1-r}$$

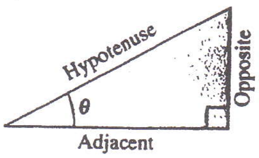
Binomial Expansion and theorem

$$(a+b)^n = \sum_{r=0}^{n} \frac{n!}{\{n-r\}!\,r!}(a^{n-r}b^r)$$

TRIGONOMETRY

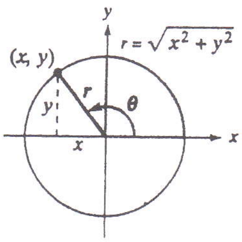
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

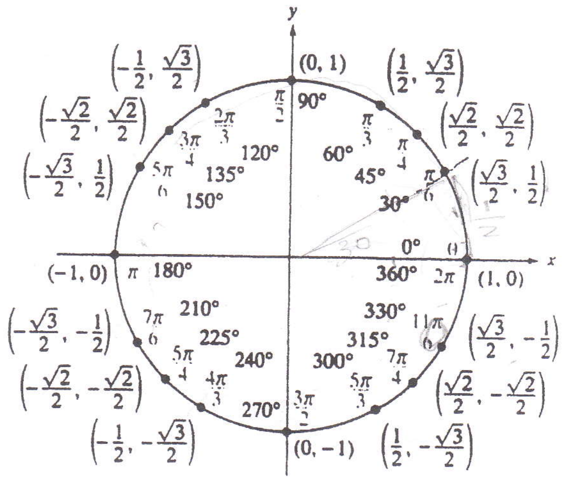


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \sec x = \frac{1}{\cos x} \qquad \tan x = \frac{1}{\cot x}$$
$$\csc x = \frac{1}{\sin x} \qquad \cos x = \frac{1}{\sec x} \qquad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$
$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \qquad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$
$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Reduction Formulas

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x$$
$$\csc(-x) = -\csc x \qquad \tan(-x) = -\tan x$$
$$\sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2\sin u \cos u$$
$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$
$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

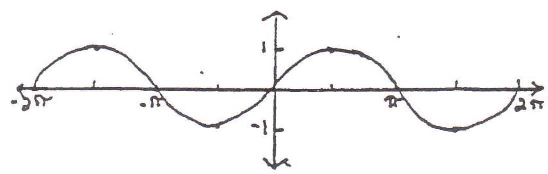
$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$
$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

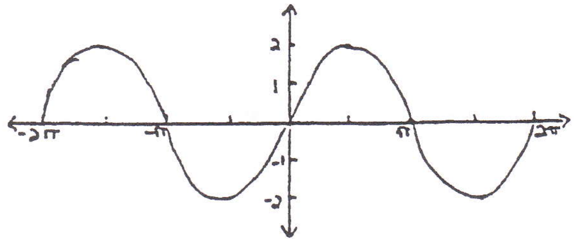
$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$
$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

Dilations are obvious in the graph of the sine function, $y = \sin x$.
Compare the graphs below.

$y = \sin x$

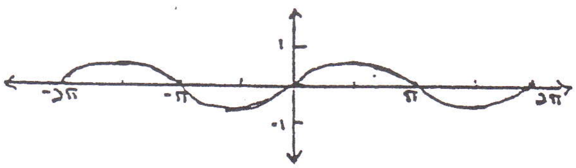


$y = 2\sin x$; $a > 1$



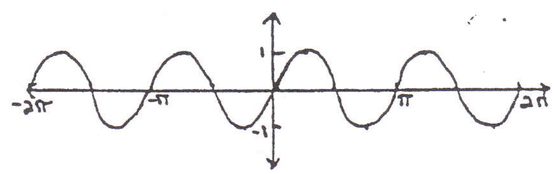
vertical stretch,
x values remain fixed,
other points move away from the x-axis

$y = \frac{1}{2}\sin x;\ \ 0 < a < 1$



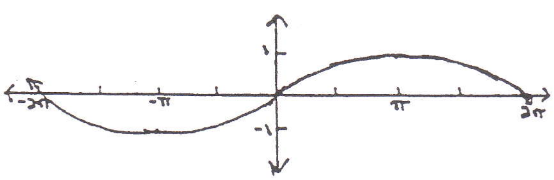
vertical shrink,
x values remain fixed,
other points move toward the x-axis

$y = \sin 2x;\ \ b > 1$



horizontal shrink,
y values remain fixed,
other points move toward y-axis

$y = \sin\frac{1}{2}x;\ \ 0 < b < 1$



horizontal stretch,
y values remain fixed,
other points move away from y-axis

ALGEBRA

Factors and Zeros of Polynomials

Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial. If $p(a) = 0$, then a is a *zero* of the polynomial and a solution of the equation $p(x) = 0$. Furthermore, $(x - a)$ is a *factor* of the polynomial.

Fundamental Theorem of Algebra

An nth degree polynomial has n (not necessarily distinct) zeros. Although all of these zeros may be imaginary, a real polynomial of odd degree must have at least one real zero.

Quadratic Formula

If $p(x) = ax^2 + bx + c$, and $0 \le b^2 - 4ac$, then the real zeros of p are $x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/2a$.

Special Factors

$x^2 - a^2 = (x - a)(x + a)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$

Binomial Theorem

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + nxy^{n-1} + y^n$$

$$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \cdots \pm nxy^{n-1} \mp y^n$$

Rational Zero Theorem

If $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients, then every *rational zero* of p is of the form $x = r/s$, where r is a factor of a_0 and s is a factor of a_n.

Factoring by Grouping

$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(ax + d)$

Arithmetic Operations

$ab + ac = a(b + c)$ $\qquad \dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$ $\qquad \dfrac{a + b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$

$\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \left(\dfrac{a}{b}\right)\left(\dfrac{d}{c}\right) = \dfrac{ad}{bc}$ $\qquad \dfrac{\left(\frac{a}{b}\right)}{c} = \dfrac{a}{bc}$ $\qquad \dfrac{a}{\left(\frac{b}{c}\right)} = \dfrac{ac}{b}$

$a\left(\dfrac{b}{c}\right) = \dfrac{ab}{c}$ $\qquad \dfrac{a - b}{c - d} = \dfrac{b - a}{d - c}$ $\qquad \dfrac{ab + ac}{a} = b + c$

Exponents and Radicals

$a^0 = 1, \quad a \neq 0$ $\qquad (ab)^x = a^x b^x$ $\qquad a^x a^y = a^{x+y}$ $\qquad \sqrt{a} = a^{1/2}$ $\qquad \dfrac{a^x}{a^y} = a^{x-y}$ $\qquad \sqrt[n]{a} = a^{1/n}$

$\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$ $\qquad \sqrt[n]{a^m} = a^{m/n}$ $\qquad a^{-x} = \dfrac{1}{a^x}$ $\qquad \sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ $\qquad (a^x)^y = a^{xy}$ $\qquad \sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

TRIGONOMETRY

Basic identities

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Addition formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

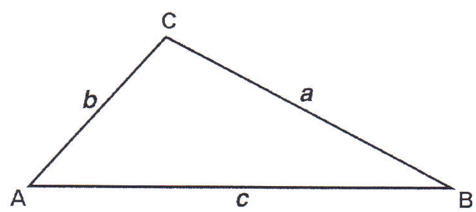
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$



PROBABILITY & STATISTICS

Permutations: $_nP_k = \dfrac{n!}{(n-k)!}$

Combinations: $_nC_k = \dfrac{n!}{k!(n-k)!}$

Sample variance = $\dfrac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}$

Finite population variance = $\dfrac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$

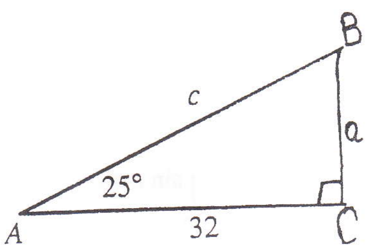
END OF DEFINITIONS AND FORMULAS

Trigonometric Functions	Right Triangle Ratios ($0° < \theta < 90°$)
$\sin\theta = \frac{y}{r}$	$\sin\theta = \frac{a}{c} = \frac{\text{length of leg opposite } \theta}{\text{length of hypotenuse}}$
$\cos\theta = \frac{x}{r}$	$\cos\theta = \frac{b}{c} = \frac{\text{length of leg adjacent to } \theta}{\text{length of hypotenuse}}$
$\tan\theta = \frac{y}{x}$	$\tan\theta = \frac{a}{b} = \frac{\text{length of leg opposite } \theta}{\text{length of leg adjacent to } \theta}$
$\csc\theta = \frac{r}{y}$	$\csc\theta = \frac{c}{a} = \frac{\text{length of hypotenuse}}{\text{length of leg opposite } \theta}$
$\sec\theta = \frac{r}{x}$	$\sec\theta = \frac{c}{b} = \frac{\text{length of hypotenuse}}{\text{length of leg adjacent to } \theta}$
$\cot\theta = \frac{x}{y}$	$\cot\theta = \frac{b}{a} = \frac{\text{length of leg adjacent to } \theta}{\text{length of leg opposite } \theta}$

Capital letters are usually used to represent the angles of triangles, or their measures. Lowercase letters refer to the sides opposite their respective angles, or to their measures. The right triangle ratios can be used to *solve* a right triangle, that is, to find the unknown measures of the sides and angles.

EXAMPLE 1 **Solve right triangle ABC if $b = 32$, $\angle A = 25°$, and $\angle C = 90°$. Fin and c to the nearest unit.**

To find a, use $\tan 25°$.



$\tan 25° = \frac{a}{32}$ — *$\tan A = \frac{a}{b}$*

$a = 32 \tan 25°$ — *Calculation-ready form*

$a = 15$ — *To the nearest unit*

To find c, use $\cos 25°$.

$\cos 25° = \frac{32}{c}$ — *$\cos A = \frac{b}{c}$*

$c = \frac{32}{\cos 25°} = 35$ — *To the nearest unit*

Since angles A and B are complementary, $\angle B = 90° - 25° = 65°$.

A **significant digit** is any nonzero digit or any zero that serves a purpose other than to locate the decimal point. Consider the following examples:

0.00*304*	Three significant digits
29.40	Four significant digits

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$

v'

7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\sin u] = (\cos u)u'$
12. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
13. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
14. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
15. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
16. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
17. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
18. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
19. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
20. $\frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1 + u^2}$
21. $\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
22. $\frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$

Basic Integration Formulas

1. $\int kf(u)\, du = k\int f(u)\, du$
2. $\int [f(u) \pm g(u)]\, du = \int f(u)\, du \pm \int g(u)\, du$
3. $\int du = u + C$
4. $\int u^n\, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u\, du = e^u + C$
7. $\int \sin u\, du = -\cos u + C$
8. $\int \cos u\, du = \sin u + C$
9. $\int \tan u\, du = -\ln|\cos u| + C$
10. $\int \cot u\, du = \ln|\sin u| + C$
11. $\int \sec u\, du = \ln|\sec u + \tan u| + C$
12. $\int \csc u\, du = -\ln|\csc u + \cot u| + C$
13. $\int \sec^2 u\, du = \tan u + C$
14. $\int \csc^2 u\, du = -\cot u + C$
15. $\int \sec u \tan u\, du = \sec u + C$
16. $\int \csc u \cot u\, du = -\csc u + C$
17. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
18. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
19. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \text{arcsec} \frac{|u|}{a} + C$

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